Binomial Theorem

Question1

If the number of terms in the binomial expansion of $(2x + 3)^{3n}$ is 22 , then the value of n is

KCET 2025

Options:

A. 8

B. 6

C. 7

D. 9

Answer: C

Solution:

A binomial expansion $(ax + b)^k$ has $k + 1$ distinct terms. Here $k = 3n$, so:

Number of terms $= 3n + 1$.

Set $3n + 1 = 22$:

$3n + 1 = 22$

$3n = 21$

$n = 7$

Answer: 7 (Option C).

Question2

The value of $^{49}C_3 + {}^{48}C_3 + {}^{47}C_3 + {}^{46}C_3 + {}^{45}C_3 + {}^{45}C_4$ **is**

KCET 2024

Options:

A. $^{50}C_4$

B. $^{50}C_3$

C. $^{50}C_2$

D. $^{50}C_1$

Answer: A

Solution:

$\because {}^{n}C_r + {}^{n}C_{r+1} = {}^{n+1}C_{r+1}$

Given, $^{49}C_3 + {}^{48}C_3 + {}^{47}C_3 + {}^{46}C_3 + {}^{45}C_3 + {}^{45}C_4$

$= {}^{49}C_3 + {}^{48}C_3 + {}^{47}C_3 + {}^{46}C_3 + {}^{46}C_4$

$= {}^{49}C_3 + {}^{48}C_3 + {}^{47}C_3 + {}^{47}C_4$

$= {}^{49}C_3 + {}^{48}C_3 + {}^{48}C_4$

$= {}^{49}C_3 + {}^{49}C_4 = {}^{50}C_4$

Question3

In the expansion of $(1+x)^n \frac{C_1}{C_0} + 2\frac{C_2}{C_1} + 3\frac{C_3}{2} + \ldots + n\frac{C_n}{C_{n-1}}$ **is equal to**

KCET 2024

Options:

A. $\frac{n(n+1)}{2}$

B. $\frac{n}{2}$

C. $\frac{n+1}{2}$

D. $3n(n+1)$

Answer: A

Solution:

$\because \frac{C_1}{C_0}+2\cdot\frac{C_2}{C_1}+3\cdot\frac{C_3}{C_2}+\ldots n\frac{C_n}{C_{n-1}}$

$=\frac{n}{1}+2\cdot\frac{\frac{n(n-1)}{1\cdot 2}}{n}+3\cdot\frac{\frac{n(n-1)(n-2)}{3\cdot 2\cdot 1}}{\frac{n(n-1)}{1\cdot 2}}+\ldots+n\cdot\frac{1}{n}$

$=n+(n-1)+(n-2)\ldots+1=\Sigma n=\frac{n(n+1)}{2}$

Question4

If n is even and the middle term in the expansion of $\left(x^2+\frac{1}{x}\right)^n$ is $924x^6$, then n is equal to

KCET 2023

Options:

A. 14

B. 12

C. 8

D. 10

Answer: B

Solution:

For $\left(x^2+\frac{1}{x}\right)^n$ have middle term n is even

$\Rightarrow n=2a; a\in N$

Middle term ${}^{2a}C_a\left(x^2\right)^a\cdot\frac{1}{x^a}={}^{2a}C_a x^a=924x^6$

$\Rightarrow \quad a=6$

Also, for $a=6, {}^{2a}C_a = {}^{12}C_6 = 924$

So, $n = 2a = 12$

Question5

The number of terms in the expansion of $(x+y+z)^{10}$ is

KCET 2020

Options:

A. 66

B. 142

C. 11

D. 110

Answer: A

Solution:

We have,

$(x+y+z)^{10}$

Total number of terms in the expansion of

$(x+y+z)^{10} = {}^{10+3-1}C_{3-1} = {}^{12}C_2 = 66$

Question6

The number of terms in the expansion of $\left(x^2+y^2\right)^{25} - \left(x^2-y^2\right)^{25}$ after simplification is

KCET 2019

Options:

A. 26

B. 0

C. 50

D. 13

Answer: D

Solution:

Key Idea. To use, if n is odd then $(x+a)^n-(x-a)^n$ has $\left(\frac{n+1}{2}\right)$ terms

Given, expansion $\left(x^2+y^2\right)^{25}-\left(x^2-y^2\right)^{25}$

Here, $n=25$ (odd)

$\therefore$ required number of terms $=\frac{25+1}{2}=\frac{26}{2}=13$

Question7

The constant term in the expansion of $\left(x^2-\frac{1}{x^2}\right)^{16}$ is

KCET 2018

Options:

A. ${}^{16}\mathrm{C}_8$

B. ${}^{16}\mathrm{C}_7$

C. ${}^{18}\mathrm{C}_9$

D. ${}^{16}C_{10}$

Answer: A

Solution:

We have, $\left(x^2-\frac{1}{x^2}\right)^{16}$

Now, $\quad T_{r+1}={}^{16}C_r\left(x^2\right)^{16-r}\left(x^{-2}\right)^r$

$\Rightarrow \quad T_{r+1} = {}^{16}C_r x^{32-2r-2r}$

T_{r+1} is constant if $32 - 2r - 2r = 0$

$\Rightarrow$ Constant term $= {}^{16}C_8$

Question8

The total number of terms in the expansion of $(x+a)^{47} - (x-a)^{47}$ after simplification is

KCET 2017

Options:

A. 96

B. 48

C. 47

D. 24

Answer: D

Solution:

The total number of terms in the expansion of $(x+a)^n - (x-a)^n$ is determined based on whether the exponent n is odd or even.

If n is odd:

Total number of terms $= \frac{n+1}{2}$

If n is even:

Total number of terms $= \frac{n}{2}$

In this specific case, we're looking at the expansion of $(x+a)^{47} - (x-a)^{47}$. Since 47 is an odd number:

Total number of terms $= \frac{47+1}{2} = \frac{48}{2} = 24$
